BIBLIOGRAPHY ON THE SIGNLESS LAPLACIAN EIGENVALUES: FIRST ONE HUNDRED REFERENCES

Dragoš Cvetković

Recall that, given a graph G, the matrix $Q = D + A$ is called the *signless Laplacian*, where A is the adjacency matrix and D is the diagonal matrix of vertex degrees. The matrix $L = D - A$ is known as the *Laplacian* of G.

Graphs with the same spectrum of an associated matrix M are called *cospectral* graphs with respect to M, or *M–cospectral* graphs. A graph H cospectral with a graph G, but not isomorphic to G, is called a *cospectral mate* of G. Let $\mathcal{G}$ be a finite set of graphs, and let $\mathcal{G}'$ be the set of graphs in $\mathcal{G}$ which have a cospectral mate in $\mathcal{G}$ with respect to M. The ratio $|\mathcal{G}'|/|\mathcal{G}|$ is called the *spectral uncertainty* of (graphs from) $\mathcal{G}$ with respect to M.

The papers [15], [16] provide spectral uncertainties r_n with respect to the adjacency matrix A, s_n with respect to the Laplacian L and q_n with respect to the signless Laplacian Q of sets of all graphs on n vertices for $n \leq 11$: the numbers q_n are smaller than the numbers r_n and s_n for $n \geq 7$. This was a strong basis for believing that studying graphs by Q–spectra is more efficient than studying them by other spectra.

This idea was accepted in [19] where it was also noted that almost no results in the literature on the spectra of signless Laplacian existed at that time. Moreover, connection with spectra of line graphs and the existence of a well developed theory of graphs with least eigenvalue -2 were used as additional arguments for studying eigenvalues of the signless Laplacian.

Next step was the paper [22] where known results were surveyed together with presentation of some new results. This paper was cited in almost all subsequent papers on the subject.

An important role in attracting many researchers played the paper [23] where thirty computer generated conjectures have been published.

Once the number of papers on the subject started to grow substantially, my colleague S. Simić and I decided to classify old and new knowledge and to sketch a theory on the signless Laplacian eigenvalues. We published the papers [41], [67], [68] with the common title "Towards a spectral theory of graphs based on the signless Laplacian".

Papers on signless Laplacian eigenvalues are continuously being published. By the end of 2010 their number has reached one hundred as the following bibliography shows.

References

Before 2002

[1] Lihtenbaum L.M., A duality theorem for simple graphs (Russian), *Usp. Mat. Nauk.* 18 (1958), No. 5. 185-190.

[2] Lihtenbaum L.M., Traces of powers of the vertex- and edge- neighbourhood matrix of a simple graph (Russian), *Izv. Viss. Ucebn. Zav. Mat.* 5 (1959), 154-163.

[3] Vahovskiĭ E.V., On the eigenvalues of the neighbourhood matrix of simple graphs (Russian), *Sibir. Mat. J.* 6 (1965), 44-49.

[4] McKay B.D., On the spectral characterization of trees, *Ars Combinatoria*, 3(1977), 219-232.

[5] Dedo E., La reconstruibilita del polinomio carrateristico del comutato di un grafo, *Boll. Unione Mat. Ital.* 18A (1981), No. 5, 423-429.

[6] Faria I., Permanental roots and the star degree of a graph, *Linear Algebra Appl.* 64 (1985), 255-265.

[7] Grone R., Merris R., Sunder V.S., The Laplacian spectrum of a graph, *SIAM J. Matrix Anal. Appl.* 11 (1990), 218-238.

[8] Desai M., Rao V., A characterization of the smallest eigenvalue of a graph, *J. Graph Theory* 18 (1994), 181-194.

[9] Grossman J.W., Kulkarni D.M., Schochetman I.E., Algebraic graph theory without orientation, *Linear Algebra Appl.*, 212/213(1994), 289-307.

[10] Cvetković D., Doob M., Sachs H., *Spectra of Graphs*, 3rd edition, Johann Ambrosius Barth Verlag, Heidelberg - Leipzig, 1995.

[11] Dam E.R. van, Graphs with few eigenvalues, Thesis, Tilburg University, Tilburg, 1996.

2002

[12] Chen Y., Properties of spectra of graphs and line graphs, *Appl. Math. J. Chinese Univ. Ser. B* 17(3) (2002), 371-376.

[13] Pan Y.-L., Sharp upper bounds for the Laplacian graph eigenvalues, *Linear Algebra Appl.*, 355(2002), 287-295.

[14] Shu J.-L., Hong Y., Wen-Ren K., A sharp upper bound on the largest eigenvalue of the Laplacian matrix of a graph, *Linear Algebra Appl.*, 347(2002), 123 - 129.

2003

[15] Dam E.R. van, Haemers W., Which graphs are determined by their spectrum?, *Linear Algebra Appl.* 373 (2003), 241-272.

2004

[16] Haemers W., Spence E., Enumeration of cospectral graphs, *Europ. J. Comb.* 25 (2004), 199-211.

[17] Das K.Ch., A characterization of graphs which ahieve the upper bound for the largest Laplacian eigenvalue, *Linear Algebra Appl.*, 376(2004), 173-186.

[18] Das K.Ch., The Laplacian spectrum of a graph, *Comput. Math. Appl.*, 48(2004), 715-724.

2005

[19] Cvetković D., Signless Laplacians and line graphs, *Bull. Acad. Serbe Sci. Arts, Cl. Sci. Math. Natur., Sci. Math.* 131(2005), No. 30, 85-92.

[20] Hong Y., Zhang X.-D., Sharp upper and lower bounds for largest eigenvalue of the Laplacian matrices of trees, *Discrete Math.*, 296(2005), 187-197.

2006

[21] Daneshgar A., Hajiabolhassan H., Graph homomorphisms and nodal domains, *Linear Algebra Appl.*, 418(2006), 44-52.

2007

[22] Cvetković D., Rowlinson P., Simić S.K., Signless Laplacians of finite graphs, *Linear Algebra Appl.*, 423(2007), No. 1, 155-171.

[23] Cvetković D., Rowlinson P., Simić S., Eigenvalue bounds for the signless Laplacian, *Publ. Inst. Math. (Beograd)*, 81(95)(2007), 11-27.

[24] Feng L., Li Q., Zhang X.-D., Minimizing the Laplacian spectral radius of trees with given matching number, *Linear Multilinear Algebra*, 55(2007), No. 2, 199-207.

[25] Feng L., Yu G., No starlike trees are Laplacian cospectral, *Univ. Beograd, Publ. Elektrotehn. Fak., Ser. Mat.*, 18(2007), 46-51.

[26] Stanić Z., *Some reconstructions in spectral graph theory and graphs with integral Q-spectrum*, (Serbian), Doctoral Thesis, Faculty of Mathematics, Belgrade, 2007.

[27] Stanić Z., There are exactly 172 connected Q–integral graphs up to 10 vertices, *Novi Sad J. Math.*, 37(2007), No. 2, 193-205.

[28] Stevanović D., Research problems from the Aveiro Workshop on Graph Spectra, *Linear Algebra Appl.*, 423(2007), No. 1, 172-181.

2008

[29] Cardoso D., Cvetković D., Rowlinson P., Simić S., A sharp lower bound for the least eigenvalue of the signless Laplacian of a non-bipartite graph, *Linear Algebra Appl.*, 429(2008), 2770-2780.

[30] Cvetković D., New theorems for signless Laplacians eigenvalues, *Bull. Acad. Serbe Sci. Arts, Cl. Sci. Math. Natur., Sci. Math.*, 137(2008), No. 33, 131-146.

[31] Fan Y.-Z., Tam B.-S., Zhou J., Maximizing spectral radius of unoriented Laplacian matrix over bicyclic graphs, *Linear Multilinear Algebra*, 56(2008), No. 4, 381-397.

[32] Liu J., Liu B., The maximum clique and the signless Laplacian eigenvalues, *Czechoslovak Math. J.*, 58(2008), No. 4, 1233-1240.

[33] Simić S.K., Stanić Z., Q–integral graphs with edge-degrees at most five, *Discrete Math.*, 308(2008), 4625-4634.

[34] Tam B.-S., Fan Y.-Z., Zhou J., Unoriented Laplacian maximizing graphs are degree maximal, *Linear Algebra Appl.*, 429(2008), 735-758.

[35] Yu G.-H., On the maximal signless Laplacian spectral radius of graphs with given matching number, Proc. Japan, Acad., Ser. A, 84(2008), 163-166.

[36] Zhu P., Wilson R.C., A study of graph spectra for comparing graphs and trees, *Pattern Recognition*, 41(2008), No. 9, 2833-2841.

[37] Zhou B., Gutman I., A connection between ordinary and Laplacian spectra of bipartite graphs, *Linear and Multilinear Algebra*, 56(2008), 305-310.

2009

[38] Akbari S., Ghorbani E., Oboudi M.R., A conjecture of square roots of Laplacian and signless Laplacian eigenvalues of graphs, arXiv:0905.2118v2 [math.CO], 6 Sep 2009

[39] Cai G.X., Fan Y.-Z., The signless Laplacian spectral radius of graphs with given chromatic number, (Chinese), *Mathematica Applicata*, 22(1)(2009),161-167.

[40] Cvetković D., Rowlinson P., Simić S., *An Introduction to the Theory of Graph Spectra*, Cambridge University Press, Cambridge, 2009.

[41] Cvetković D., Simić S.K., Towards a spectral theory of graphs based on the signless Laplacian, I, *Publ. Inst. Math.(Beograd)*, 85(99)(2009), 19-33.

[42] Dam E.R. van, Haemers W., Developments on spectral characterization of graphs, *Discrete Math.*, 309(2009), No. 3, 576-586.

[43] Das K.Ch., A sharp upper bound on the maximal entry in the principal eigenvector of symmetric nonnegative matrix, *Linear Algebra Appl.*, 431(2009), 1340-1350.

[44] Fan Y.-Z., Yang D., The signless Laplacian spectral radius of graphs with given number of pendant vertices, *Graphs and Combinatorics*, 25(2009), 291-298.

[45] Feng L., Yu G., The signless Laplacian spectral radius of unicyclic graphs with graph constraints, *Kyungpook Math.*, 49(2009), 123-131.

[46] Feng L.-H., Yu G.-H., On three conjectures involving the signless Laplacian spectral radius of graphs, *Publ. Inst. Math.(Beograd)*, 85(99)(2009), 35-38.

[47] Freitas M.A.A. de, Del-Vecchio R.R., Abreu N.M.M. de, Kirkland S., On Q-spectral integral variation, *Electronic Notes in Discrete Math.*, 35(2009), 203-208.

[48] Gutman I., Kiani D., Mirzakhah M., Zhou B., On incidence energy of a graph, *Linear Algebra Appl.*, 431(2009), 1223-1233.

[49] Haemers W., Regularity and spectra of graphs, *Surveys in Combinatorics 2009*, ed. S. Huchynska, J.D. Mitchel, C.M. Roney-Dougal, Cambridge University Press, Cambridge, 2009, 75-90.

[50] Hansen P., Lucas C., An inequality for the signless Laplacian index of a graph using the chromatic number, *Graph Theory Notes New York*, 57(2009), 39-42.

[51] Jooyandeh M.R., Kiani D., Mirzakhah M., Incidence energy of a graph, *MATCH Commun. Math. Comput. Chem.*, 62(2009), 561-572.

[52] Omidi G.R., On a signless Laplacian spectral characterization of T-shape trees, *Linear Algebra Appl.* 431(2009), 1607-1615.

[53] Rojo O., Spectra of copies of a generalized Bethe tree attached to any graph, *Linear Algebra Appl.*, 431(2009), 863-882.

[54] Simić S.K., Stanić Z., On some forests determined by their Laplacian or signless Laplacian spectrum, *Comput. Math. Appl.*, 58(2009), 171-178.

[55] Stanić Z., Some results on Q-integral graphs, *Ars Combin.*, 90(2009), 321-335.

[56] Wang J., Huang Q., Belardo F., Li Marzi E.M., On graphs whose signless Laplacian index does not exceed 4.5, *Linear Algebra Appl.*, 431(2009), 162-178.

[57] Wu R.R., Fan Y.-Z., The signless Laplacian spectral radius of graphs with given number of cut edges, *J. Anhui Univ. Sci. Technology (Nat.Sci.)*, 29(2)(2009), 66-69.

[58] Zhang X.-D., The signless Laplacian spectral radius of graphs with given degree sequences, *Discrete Appl. Math.*, 157(2009), 2928-2937.

[59] Zhang Y., Liu X., Zhang B., Yong X., The lollipop graph is determined by its Q–spectrum, *Discrete Math.*, 309(2009), 3364-3369.

2010

[60] Akbari S., Ghorbani E., Koolen J.H., Oboudi M.R., A relation between the Laplacian and signless Laplacian eigenvalues of a graph, *J. Albebraic Combinatorics*, 32(2010), No. 3, 459-464.

[61] Akbari S., Ghorbani E., Koolen J.H., Oboudi M.R., On sum of powers of the Laplacian and signless Laplacian eigenvalues of graphs, *The Electronic J. Combinatorics*, 17(2010), #R115

[62] Andjelić M., Simić S.K., Some notes on the threshold graphs, *Discrete Math.*, 310(2010), 2241-2248.

[63] Aouchiche M., Hansen P., A survey of automated conjectures in spectral graph theory, *Linear Algebra Appl.* 432(2010), 2293-2322.

[64] Belardo F., Li Marzi E.M., Simić S.K., Wang J.F., On the index of necklaces, *Graphs Combin.*, 26(2010), No. 2, 163-172.

[65] Chang T.-J., Tam B.-S., Graphs with maximal signless Laplacian spectral radius, *Linear Algebra Appl.* 432(2010), 1708-1733.

[66] Chen Y., Wang L., Sharp bounds for the largest eigenvalue of the signless Laplacian of a graph, *Linear Algebra Appl.*, 433(2010), 908-913.

[67] Cvetković D., Simić S.K., Towards a spectral theory of graphs based on the signless Laplacian, II, *Linear Algebra Appl.*, 432(2010), 2257-2272.

[68] Cvetković D., Simić S.K., Towards a spectral theory of graphs based on the signless Laplacian, III, *Appl. Anal. Discrete Math.*, 4(2010), 156-166.

[69] Cvetković D., Simić S.K., Stanić Z., Spectral determination of graphs whose components are paths and cycles, *Comput Math. Appl.*, 59(2010), 3849-3857.

[70] Das K.Ch., On conjectures involving second largest signless Laplacian eigenvalue of graphs, *Linear Algebra Appl.*, 432(2010), 3018-3029.

[71] Feng L., The signless Laplacian spectral radius for bicyclic graphs with k pendant vertices, *Kyungpook Math. J.*, 5(2010), 109-116.

[72] Feng L., Yu G. Ilić A., The Laplacian spectral radius for unicyclic graphs with given independence number, *Linear Algebra Appl.*, 433(2010), 934-944.

[73] Freitas M.A.A. de, Abreu N.M.M. de, Del-Vecchio R.R., Jurkiewicz S., Infinite families of Q–integral graphs, *Linear Algebra Appl.*, 432(2010), 2352-2360.

[74] Geng X., Li S., Simić S.K., On the spectral radius of quasi-k-cyclic graphs, *Linear Algebra Appl.*, 433(2010), 1561-1572.

[75] Gutman I., Robbiano M., Martins E.A., Cardoso D.M., Medina L., Rojo O., Energy of line graphs, *Linear Algebra Appl.*, 433(2010), 1312-1323.

[76] Hansen P., Lucas C., Bounds and conjectures for the signless Laplacian index of graphs, *Linear Algebra Appl.*, 432(2010), 3319-3336.

[77] Li R., Shi J., The minimum signless Laplacian spectral radius of graphs with given independence number, *Linear Algebra Appl.*, 433(2010), 1614-1622.

[78] Liu M., Liu B., The signless Laplacian spread, *Linear Algebra Appl.*, 432(2010), 505-514.

[79] Liu M., Tan X., Liu B., The (signless) Laplacian spectral radius of unicyclic and bicyclic graphs with n vertices and k pendant vertices, *Czechoslovak Math. J.*, 60(2010), No. 3, 849-867 .

[80] Mirzakhah M., Kiani D., The sun graph is determined by its signless Laplacian spectrum, *Electronic J. Linear Algebra*, 20(2010), 610-620.

[81] Oliveira C.S., Lima L.S. de, Abreu N.M.M. de, Hansen P., Bounds on the index of the signless Laplacian of a graph, *Discrete Appl. Math.*, 158(2010), No. 4, 355-360.

[82] Oliveira C.S., Lima L.S. de, Abreu N.M.M. de, Kirkland S., Bounds on the Q–spread of a graph, *Linear Algebra Appl.*, 432(2010), 2342-2351.

[83] Omidi G.R., Vatandoost E., Starlike trees with maximum degree 4 are determined by their signless Laplacian spectra, *Electronic J. Linear Algebra*, 20(2010), 274-290.

[84] Rojo O., Medina L., Spectral characterization of some weighted rooted graphs with cliques, *Linear Algebra Appl.*, 433(2010), 1388-1409.

[85] Simić S.K., Stanić Z., On Q-integral $(3, s)$-semiregular bipartite graphs, *Appl. Anal. Discrete Math.*, 4(2010), 167-174.

[86] Tam B.-S., Wu S.-H., On the reduced signless Laplacian spectrum of a degree maximal graph, *Linear Algebra Appl.*, 432(2010), 1734-1756.

[87] Tan S.-W., On the Laplacian spectral radius of weighted trees with a positive weight set, *Discrete Math.*, 310(2010), 1026-1036.

[88] Tan S.-W., On the weighted trees with given degree sequence and positive weight set, *Linear Algebra Appl.*, 433(2010), 380-389.

[89] Wang J.-F., Belardo F., Huang Q., Borovićanin B., On the two largest Q-eigenvalues of graphs, *Discrete Math.*, 310(2010), 2858-2866.

[90] Wang J.-F., Belardo F., Huang Q., Li Marzi E.M., Spectral characterizations of dumbbell graphs, *Electronic J. Combinatorics*, 17(2010) #R42.

[91] Wang J.-F., Huang Q., An X.H., Belardo F., Some results on the signless Laplacians of graphs, *Appl. Math. Letters*, 23(2010), No. 9, 1045-1049.

[92] Wang J.-F., Huang Q., Belardo F., Li Marzi E.M., On the spectral characterizations of ∞-graphs, *Discrete Math.*, 310(2010), No. 13/14, 1845-1855.

[93] Ye M.-L., Fan Y.-Z., Wang H.-F., Maximizing signless Laplacian or adjacency spectral radius of graphs subject to fixed connectivity, *Linear Algebra Appl.*, 433(2010), 1180-1186.

[94] Zhou B., Signless Laplacian spectral radius and Hamiltonicity, *Linear Algebra Appl.*, 432(2010), 566-570.

[95] Zhu B.-X., On the signless Laplacian spectral radius of graphs with cut vertices, *Linear Algebra Appl.*, 433(2010), 928-933.

In process of publication

[96] Chang T.-C., Tam B.-S., Wu S.-H., Theorems on partitioned matrices revisited and their applications to study of graph spectra, *Linear Algebra Appl.*, to appear.

[97] Feng L.-H., Yu G.-H., The signless Laplacian spectral radius of graphs with given diameter, *Utilitas Math.*, 83(2010), to appear.

[98] Gong S.-C., Fan Y.-Z., Yin Z.-X., The unicyclic graphs with extremal signless Laplacian spectral spread, *Appl. Math. J. Chin. Univ.*, to appear.

[99] Liu M.H., Liu L.B., On the signless Laplacian spectral radii of bicyclic and tricyclic graphs, *Ars Combinatoria*, to appear.

[100] Yu G., Wu Y., Shu J., Sharp bounds on the signless Laplacian spectral radius of graphs, *Linear Algebra Appl.*, to appear.